5-5 Videos Guide

5-5a

- Average value of a function f over an interval $[a, b]$
    - $f_{ave} = \frac{1}{b-a} \int_a^b f(x)\, dx$

Exercise:

- Find the average value of the function on the given interval.

$g(t) = \frac{t}{\sqrt{3+t^2}}, \quad [1,3]$

Theorem (statement):

- The Mean Value Theorem for Integrals: If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that

$$f(c) = f_{avg} = \frac{1}{b-a} \int_a^b f(x)\, dx$$

that is, $\int_a^b f(x)\, dx = f(c)(b - a)$

5-5b

Exercises:

- (a) Find the average value of f on the given interval.

    (b) Find c in the given interval such that $f_{avg} = f(c)$.

    (c) Sketch the graph of f and a rectangle whose base is the given interval and whose area is the same as the area under the graph of f.

    $f(x) = \sqrt[3]{x}, \quad [0,8]$

- Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.